Cellular Models

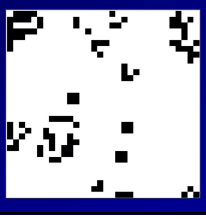
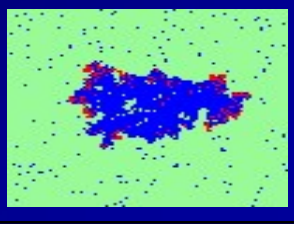
Exploring Cellular Automata and Other Cellular Models

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UNIQ Programme, 2021

1

Shells, “Life”, Epidemics, Evolution ...



2

2

Cellular Automata

- Cellular Automata are simple models in which we treat the individuals as located on a geometrical grid (most commonly rectangular), with simple interactions taking place that control the change of configuration from one “generation” to the next.
- They give a powerful illustration of how complex high-level behaviour can *emerge* from simple low-level rules, often in ways which would defy any attempt to understand the high-level phenomena without the possibility of computational “experiments” and observation.

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Conway’s “Game of Life”

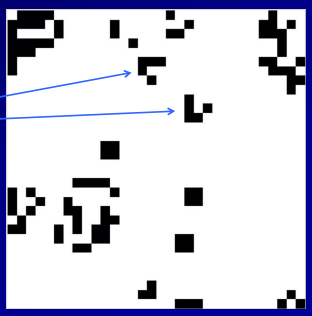
- Each cell is the centre of a square neighbourhood, with eight neighbours, and is either alive or dead. From each “generation” to the next, the pattern of live and dead cells changes as follows:
  - If a live cell has either 2 or 3 live neighbours, it survives; otherwise it dies.
  - If a dead cell has exactly 3 live neighbours, then it becomes alive.
- These simple rules give rise to complex patterns, sufficiently powerful to permit the construction (given space and patience!) of a full-scale computer – see Poundstone, *The Recursive Universe* (1985).

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Potential for Complex Structure

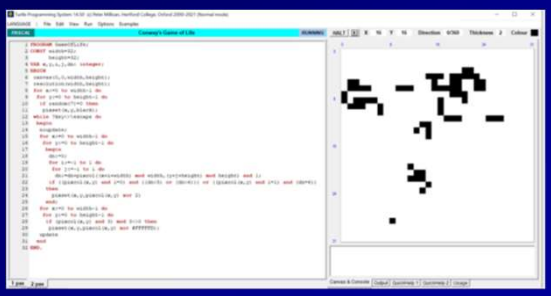
Here we see two gliders, a 5-pixel pattern that moves diagonally



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Programming Cellular Automata in *The Turtle System*

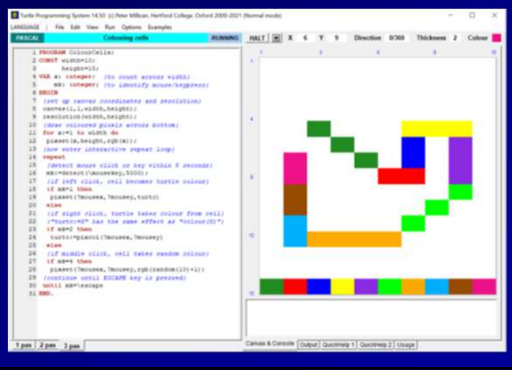


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Cellular Models

Making the Canvas Cellular



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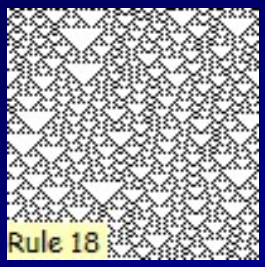
Hiding Information in Pixels

- Programming the Game of Life is tricky, because all of the cells have to be updated *synchronously* (i.e. at the same time), so as we work through the cells updating them, we need to be able to store *both* the updated status of each of them, and also its previous status (since this affects its neighbours).
- There are neat ways of doing this using the coloured pixels to store both types of information – see the accompanying handout for details. Hiding information in pixels is a form of *steganography*.
- One-dimensional cellular automata are easier to program, because each generation can be displayed horizontally, with the next generation following below.

8

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Intriguing Echoes of Natural Patterns



One-Dimensional CA



Conus textile shell

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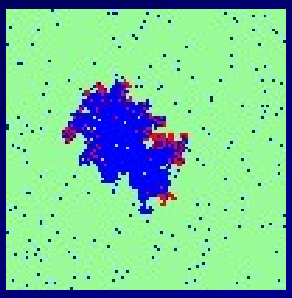
Cellular Models

- Cellular *automata* are usually understood as proceeding entirely automatically from one generation to the next, following fixed and deterministic rules.
- But cellular models more generally can work in many different ways, and are often easier to program because they can proceed *asynchronously* – taking one cell at a time, which is often chosen *randomly*.

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Modelling Epidemic Spread



Green:
susceptible

Red:
infectious

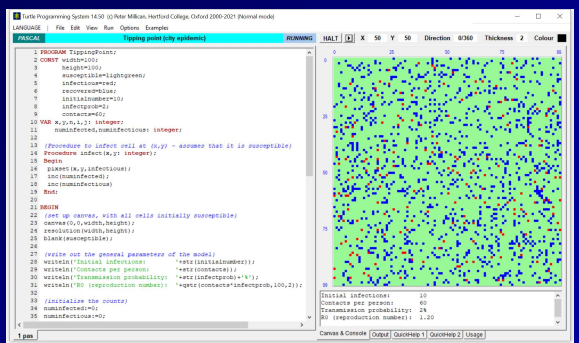
Blue:
immune
(either naturally, or after recovery)

(Known as the SIR – Susceptible / Infectious / Recovered – epidemic model.)

11

11

Modelling a "Tipping Point"



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12

Peter Millican, Hertford College, Oxford, July 2021

Cellular Models

Evolutionary Models

- Evolution operates by inherited variation together with *natural selection* based on competition - those inherited traits which bestow longer life and greater fertility on the organisms that carry them "select themselves" by being preferentially copied from generation to generation. Thus evolution is not "purposeful" and cannot "look ahead".
- Traits are selected purely in terms of their effects, which can be indirect and highly complex. Computer models can provide a valuable tool for understanding these things.

13

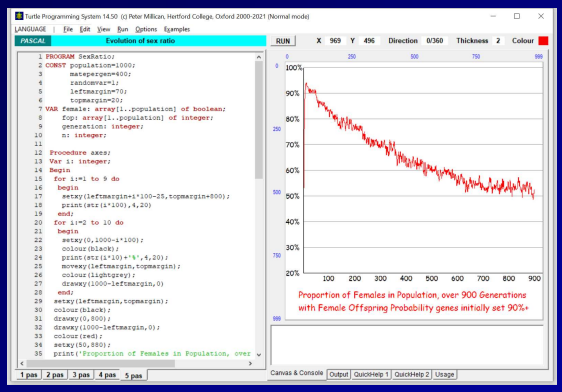
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Example: The Sex Ratio in Mammals

- A puzzle dating back to the 18th century:
  - Why is the ratio of males to female births close to 1:1 in humans, dogs, cattle, … ? Bulls mate with multiple cows, and don't help with the offspring, so why does nature produce so many bulls?
  - This wasn't explained until 1930! In a population with more cows than bulls, those individuals giving birth to males will have more *grandchildren*, so the genes that favour male over female offspring will spread until the proportion becomes close to 50:50.
    - *The Turtle System*'s "Evolution of Sex Ratio" model illustrates exactly this dynamic …

14

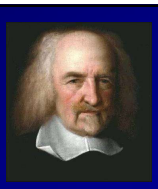
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15

Modelling Society



- Thomas Hobbes was the first modern thinker to try to give a "scientific" explanation of how society arises. He thought that only by agreeing together to obey an absolute ruler can we avoid descending into a life-threatening war of "all against all".
- The difficulty of achieving cooperation *without* some authority that forces us to obey common rules is illustrated by the famous "Prisoner's Dilemma":

	You cooperate	You defect
I cooperate	**3** *(Reward)*	**0** *(Sucker's Payoff)*
I defect	**5** *(Temptation)*	**1** *(Punishment)*

16

16

The Prisoner's Dilemma and Evolution

- In the Prisoner's Dilemma, the only "self-interestedly rational" action seems to be to defect. But it's called a "dilemma" because if both parties follow self-interest in this way, they both do *worse* than if they had "altruistically" cooperated!
- Hobbes was far too pessimistic about human nature: we don't in fact always act for our self-interest, nor only for the benefit of our family and friends (see http://www.philocomp.net/videos/general_lectures.htm Lecture 6 if you need convincing).
- But this raises a puzzle: how can *cooperative* and even *altruistic* behaviour evolve, if evolution operates by natural selection on favourable outcomes?

17

17

Iterating the Prisoner's Dilemma

- In nature, we encounter each other repeatedly, and we follow *patterns* of behaviour rather than calculating every decision separately.
- What happens if we allow the interactions to be *iterated* (i.e. repeated), say 10 times, so that individuals can condition their responses in the light of previous interactions?
- Suppose we also put this in an evolutionary or other competitive context, where behaviours can evolve, or be learned or transmitted between different individuals?

18

18

Cellular Models

Simple Strategies in the Iterated PD

- The "Iterated Prisoner's Dilemma" involves extended ***strategies*** of interaction rather than individual events of cooperation and/or defection.
- Some examples of very simple strategies:
  - ALLD (Always Defect)
  - ALLC (Always Cooperate)
  - ALTCD (Alternately Cooperate then Defect)
  - TFT (Tit-for-Tat: start by Cooperating, then respond in kind to partner's last action)
  - TF2T (Tit-for-Two-Tats: Defect only immediately after partner has defected twice in a row)
  - GRIM (Start by Cooperating, but if partner ever defects, continuously Defect from then onwards)

19

19

How the Strategies Square Off

- Here is how the six strategies (listed in order of increasing "niceness") fare in a sequence of 10 interactions against each of the others. The first score in each pair goes to the "row" strategy, and the second score to the "column" strategy.

	ALLD	ALTCD	GRIM	TFT	TF2T	ALLC
ALLD	10/10	30/5	14/9	14/9	18/8	50/0
ALTCD	5/30	20/20	12/27	28/23	40/15	40/15
GRIM	9/14	27/12	30/30	30/30	30/30	30/30
TFT	9/14	23/28	30/30	30/30	30/30	30/30
TF2T	8/18	15/40	30/30	30/30	30/30	30/30
ALLC	0/50	15/40	30/30	30/30	30/30	30/30

20

20

Evolving Strategies in the IPD

- The IPD can be used to test not only how different strategies perform against each other in a static context, but also how the strategies would multiply or decline in an "ecological" context (where future frequency depends on past performance).
- A further extension of this idea would be to model the evolution of strategies (with mutation and/or recombination as well as variation in frequency).
- *Turtle* contains a very simple ecological model involving ALLC, ALLD and TFT, with each cell partnering each of its neighbours for 10 iterations. For each cell in the next generation, the most successful neighbour "moves in".

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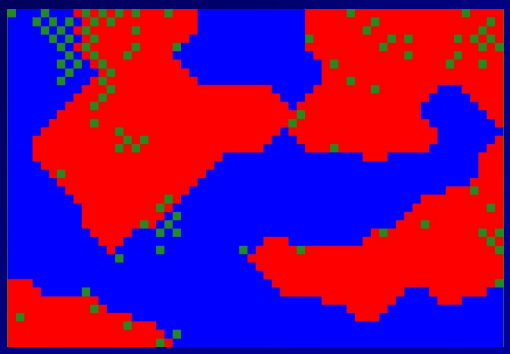
Well-Known Results from the IPD

- In Robert Axelrod's first organised computer tournament (published 1980), TFT won but TF2T would have won had it been entered. The "nice" strategies (those which never defect first) came out ahead of the "nasty" ones.
- TFT also won Axelrod's second tournament, and a third "ecological" tournament in which the makeup of each generation depended on previous performance.
- TFT does less well if "noise" occurs – in this case a "generous" version which probabilistically forgives after a defection – GTFT – can eventually displace it.
- The "Pavlov" or "Win-stay/Lose-shift" strategy also does well in a probabilistic environment. But there is no strategy which does best in all circumstances.

22

22

Schelling's Segregation Model



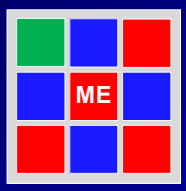
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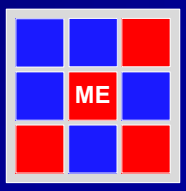
How the Model Works

A 60 x 40 grid with 100 random green spaces, but blues and reds evenly distributed everywhere else. Individuals are "unhappy" – and therefore move – if the other colour has a majority of 2 or more amongst their neighbours:

Happy:



Unhappy:



24

24

Peter Millican, Hertford College, Oxford, July 2021